The Isopycnal Diffusivity Tensor Based on Lagrangian Particle Statistics

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Objective

- Understand transport mechanisms at the mesoscale by analysing Lagrangian particle statistics of an eddy-permitting model of the North Atlantic with 10km horizontal resolution.
- Search for a reasonable estimate of spatially varying isopycnal eddy diffusivity.

Lagrangian Method

Diffusivity Tensor

- Uniform release, 120-day pseudo-trajectories in 5.5 years of model velocity
- Efficient sampling:
  - One statistically independent float observed per month per bin: 70 floats in bin
- Horizontal velocity and displacement statistics (v' and r', respectively) yield isopycnal diffusivity tensor κ_{jk}:

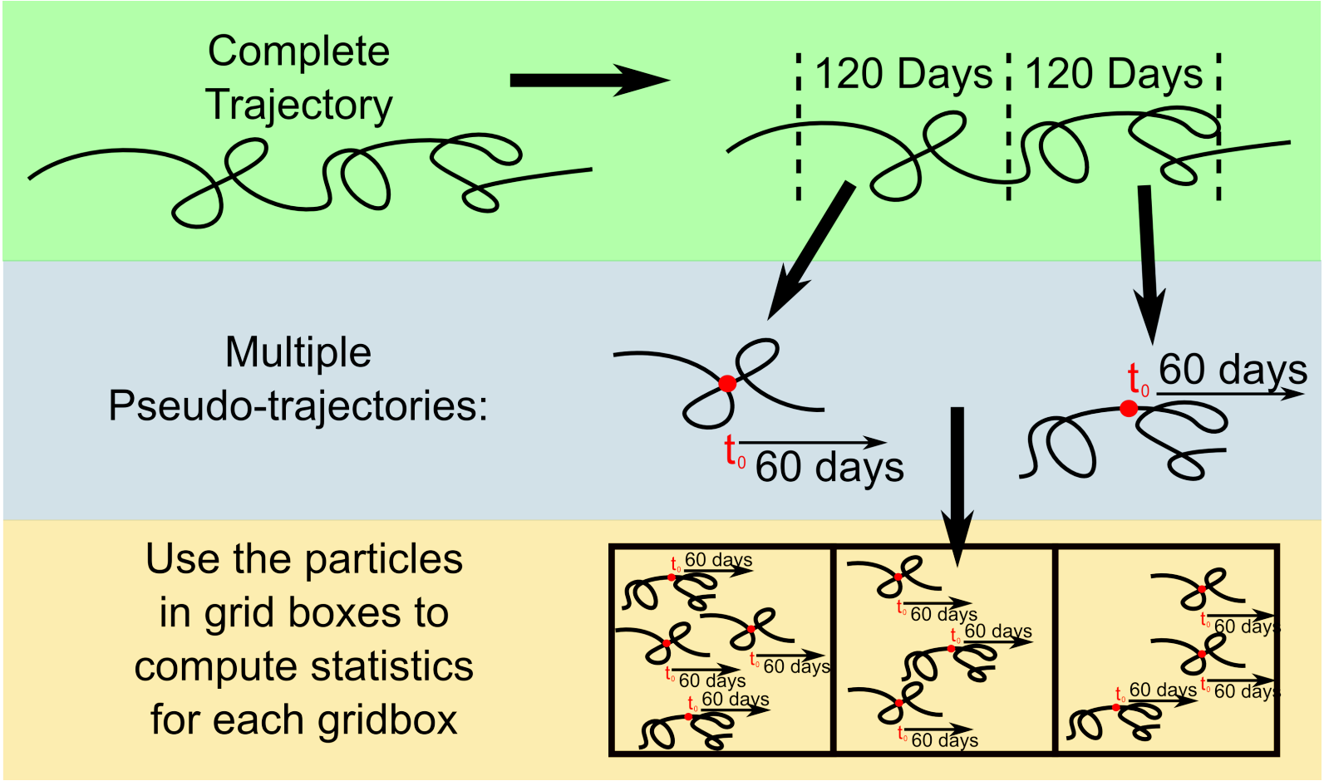
$$\kappa_{jk}(\mathbf{x}, t) = -\langle v'_j(t_0|\mathbf{x}, t_0) r'_k(t_0 - t|\mathbf{x}, t_0)\rangle$$
$$\kappa_{jk} = \kappa_{jk}^{sym} + \kappa_{jk}^{asym}$$

- Physical Interpretation of κ_{jk}^{sym}: Time derivative of negative displacement covariance tensor. $\kappa_{jk}^{sym} \simeq -2\frac{d}{dt}\langle r'^2_{ij}\rangle$

K_{ij}^{sym} Convergent ⟷ Large-eddy induced random walk ⟷ "***Diffusive***" Material Transport

- Principal axis transformation of κ_{jk}^{sym}:

$$\kappa_{jk}^{sym} = Q \begin{pmatrix} \kappa_{maj}^{sym} & 0 \\ 0 & \kappa_{min}^{sym} \end{pmatrix} Q^T$$



Complete Trajectory

120 Days | 120 Days

Multiple Pseudo-trajectories:

Use the particles in grid boxes to compute statistics for each gridbox

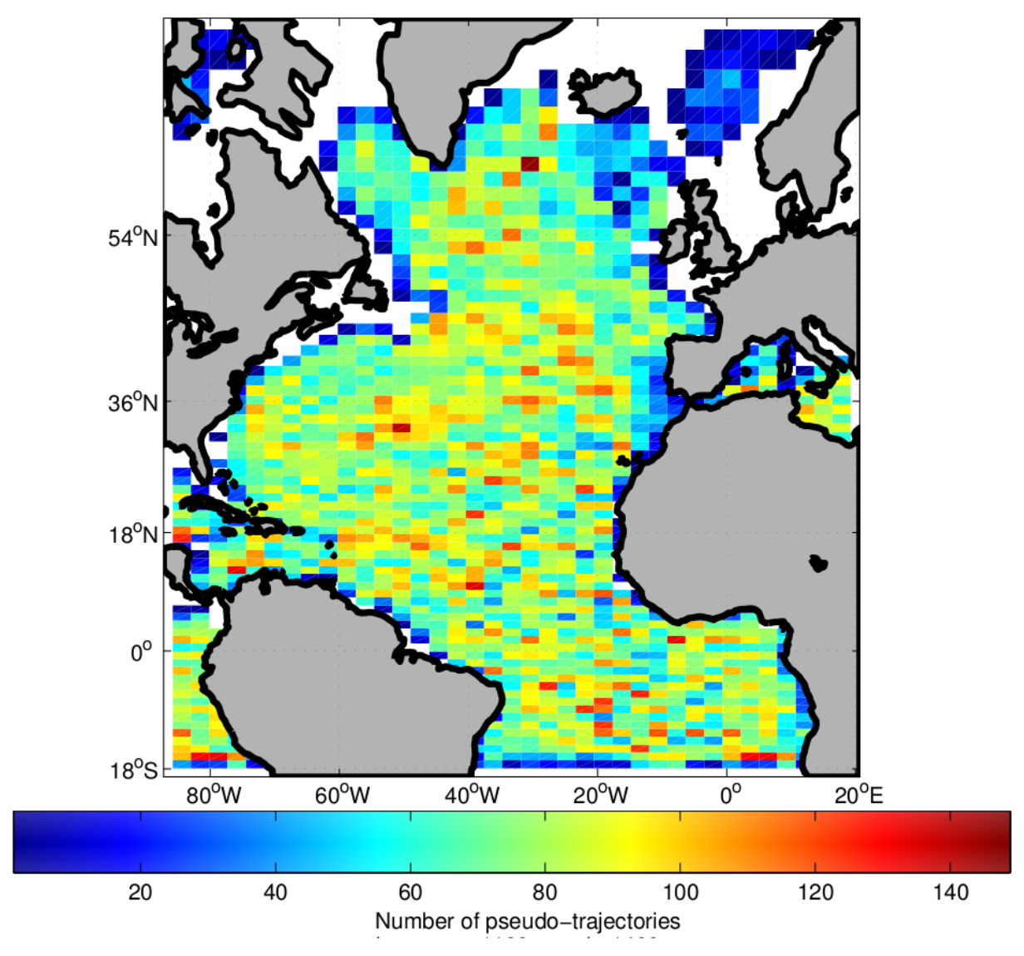


Figure: Number of pseudo-trajectories in grid box. Plotted gridboxes are 300m thick and centered at 1250m depth.

Comparison of κ_{jk}^{sym} from Model and Observations

- Estimates of κ_{jk}^{sym} agree in amplitude and pattern, but maximum of North Atlantic Current is shifted southward in the model

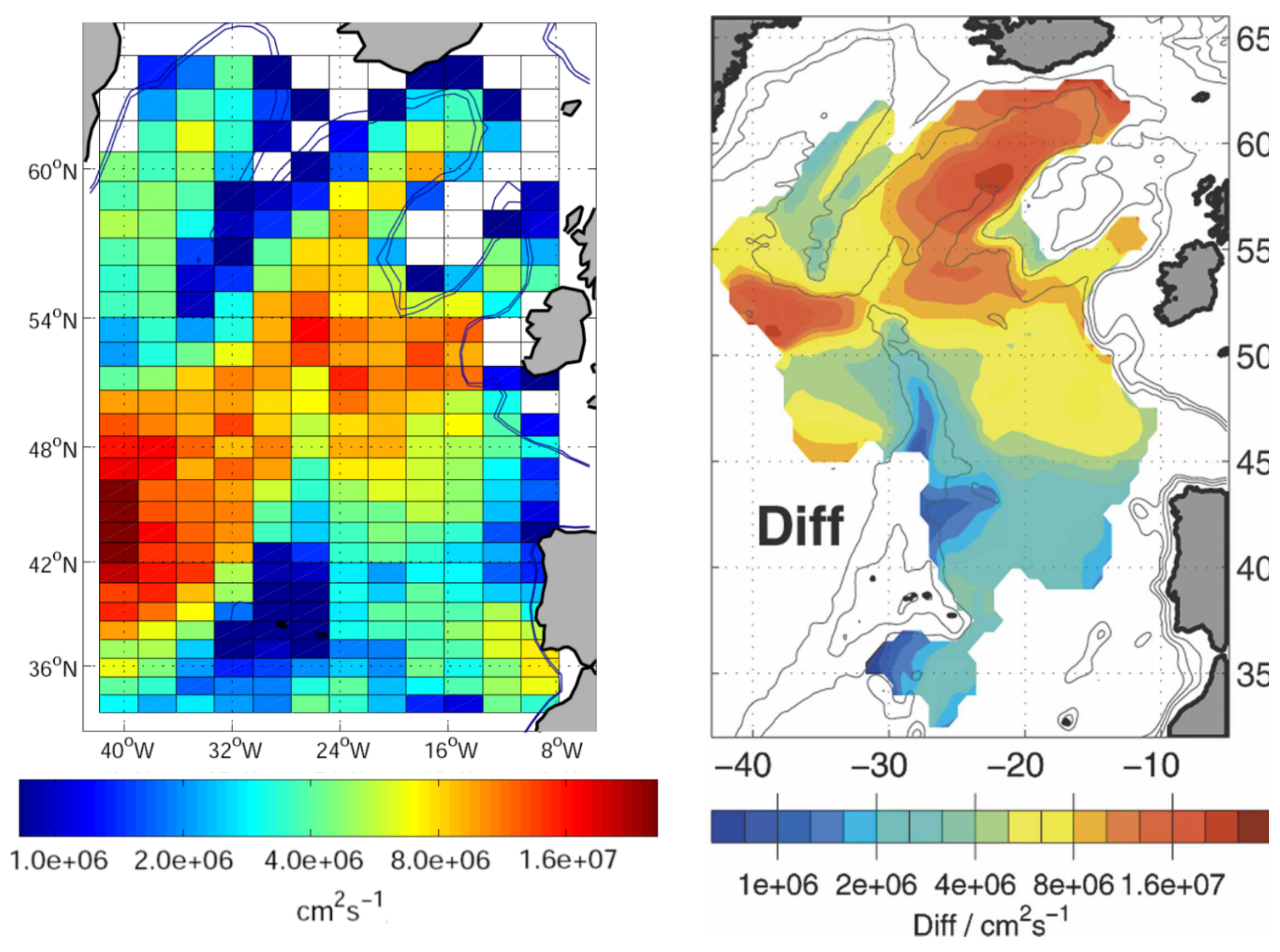
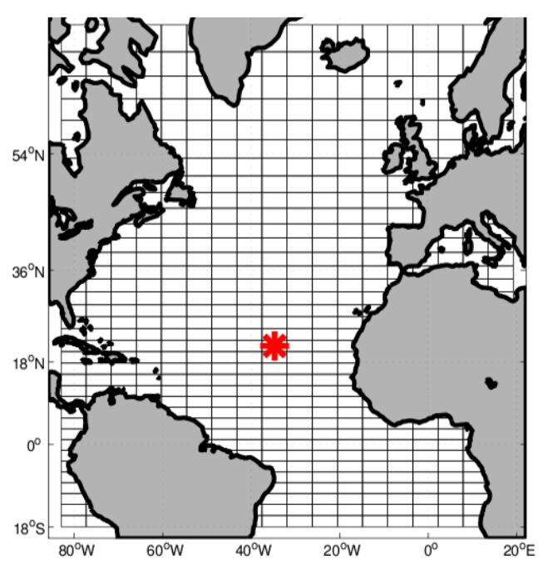


Figure: **Left:** Maximum of minor principal component $max(\kappa_{min}^{sym})$ of the diffusivity tensor in a region of the Northeastern Atlantic Ocean, during the first 60 days after deployment. Gridboxes are bounded by the 1500m and 1750m isobaths. **Right:** Diffusivity calculated from observed floats by Lankhorst and Zenk (2006). Depicted is the value of $\overline{u'^2}T$, where $\overline{u'^2}$ is velocity variance and T is the velocity autocorrelation, integrated up to the first zero crossing.

Interpretation of Symmetric Part κ_{jk}^{sym}: A Case Study



We look at the timeseries of the minor principle component in a single gridbox.

- Interpretation of $\lim_{t\to\infty} \kappa_{jk}^{sym}$: Turbulent diffusivity within a mean isopycnal
- Uniform Release: Minor principal component of κ_{jk}^{sym}: insensitive to mean flow shear (Oh et. al. 2000)

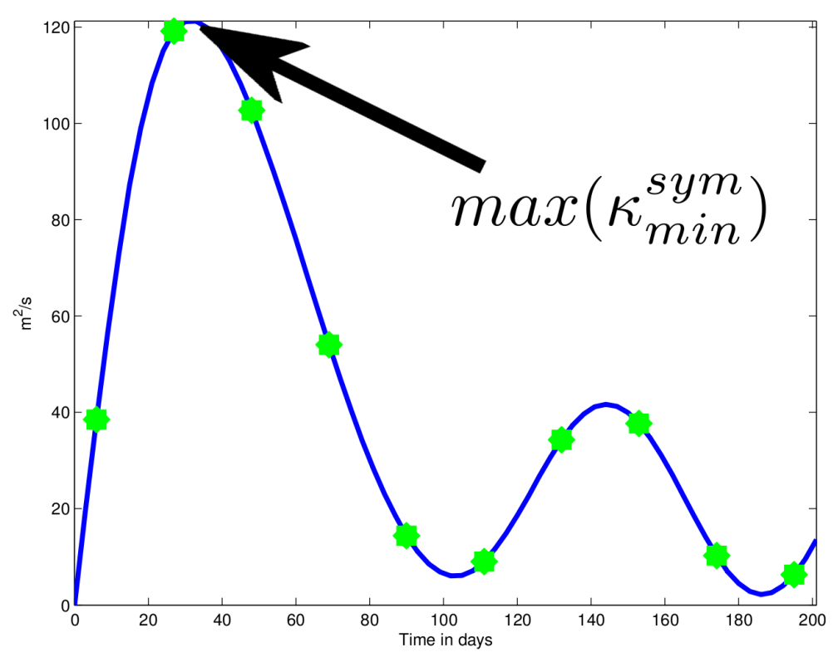


Figure: **Left:** Minor principle component of κ_{jk}^{sym}, in a gridbox between 2000m and 2300m depth. Green dots are points in time shown below.

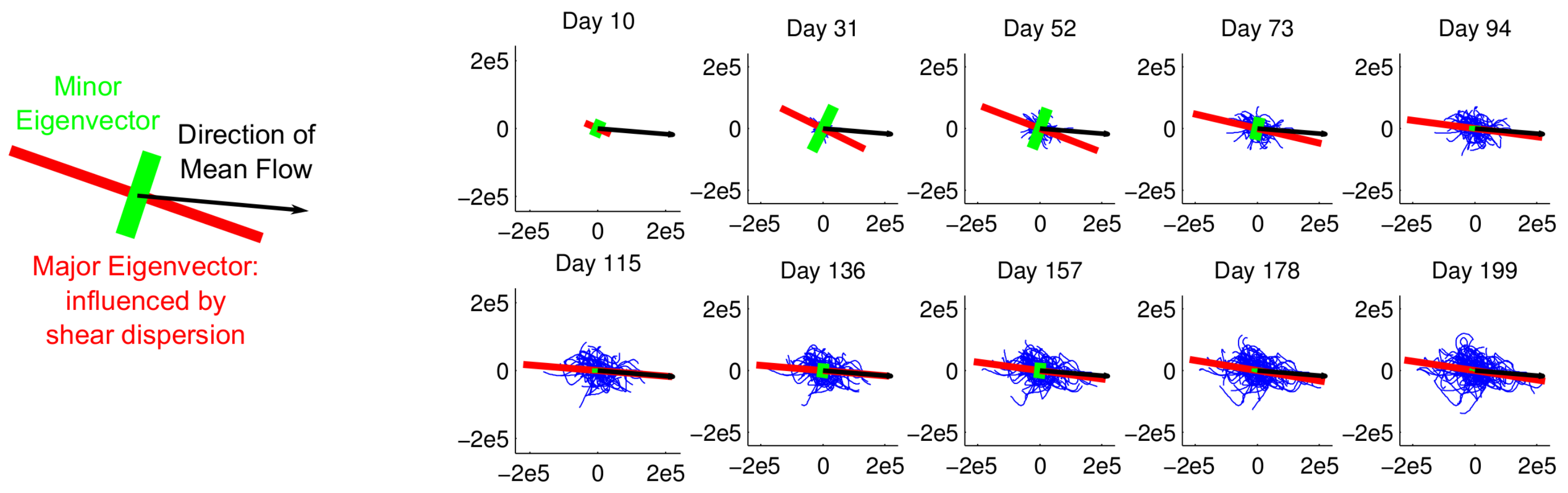


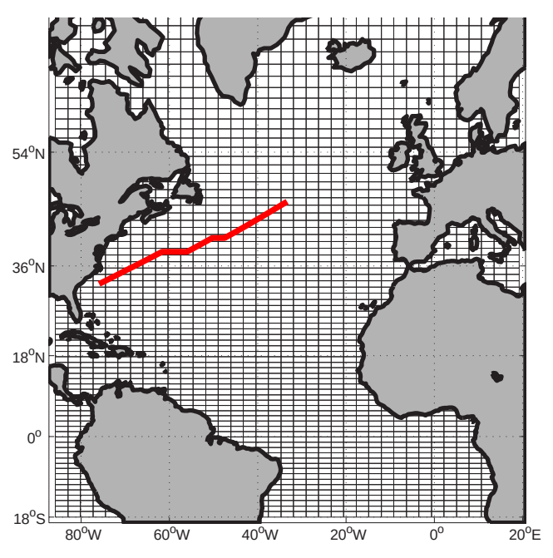
Figure: Evolution of κ_{jk}^{sym} with time. The major eigenvector progressively rotates and aligns with direction of mean flow. Drifter trajectories are plotted in blue.

- Our method yields timeseries of the minor principle component that do not converge in most gridboxes for up to 200 days after deployment

Summary: For our averaging method, κ_{min}^{sym} is insensitive to sheared mean flow. κ_{min}^{sym} oscillates and does not converge within gridboxes.

Vertical Structure of κ_{min}^{sym} in the North Atlantic Current

The North Atlantic Current is associated with high eddy kinetic energy and we expect that diffusivity may have a pronounced vertical structure there. The figures indicate that



- $max(\kappa_{min}^{sym})$ and EKE are both surface intensified and decrease with depth.
- The timelag at which κ_{min}^{sym} obtains its maximum may be proportional to the average eddy turnaround time.
- We plot the values of $max(\kappa_{min}^{sym})$ at a lag of 60 days to demonstrate that our method is not capable of revealing a possible convergence in every gridbox at this lag.

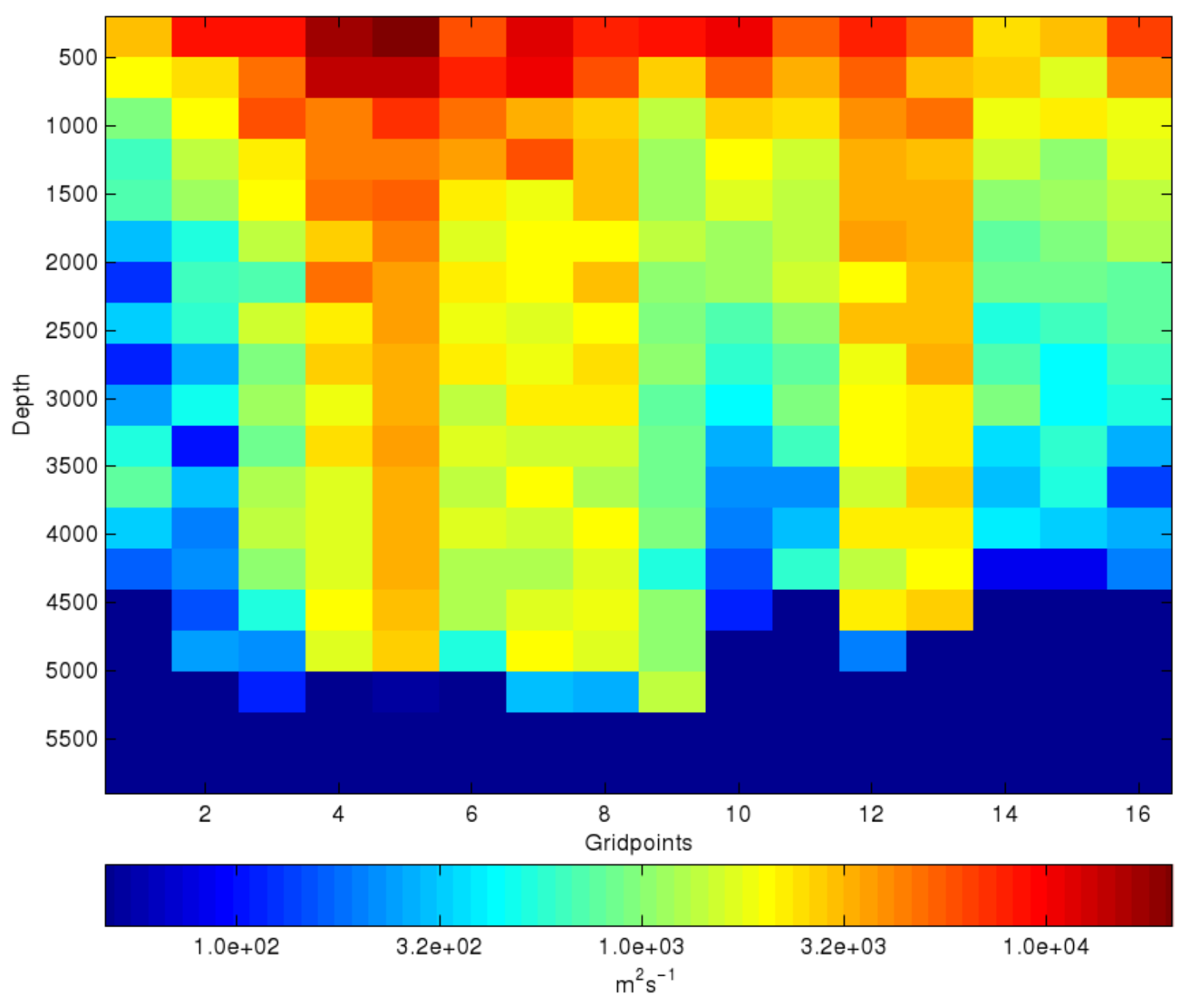


Figure: Maximum of minor principal component whithin the first 60 days after deployment along the NAC (red line on map).

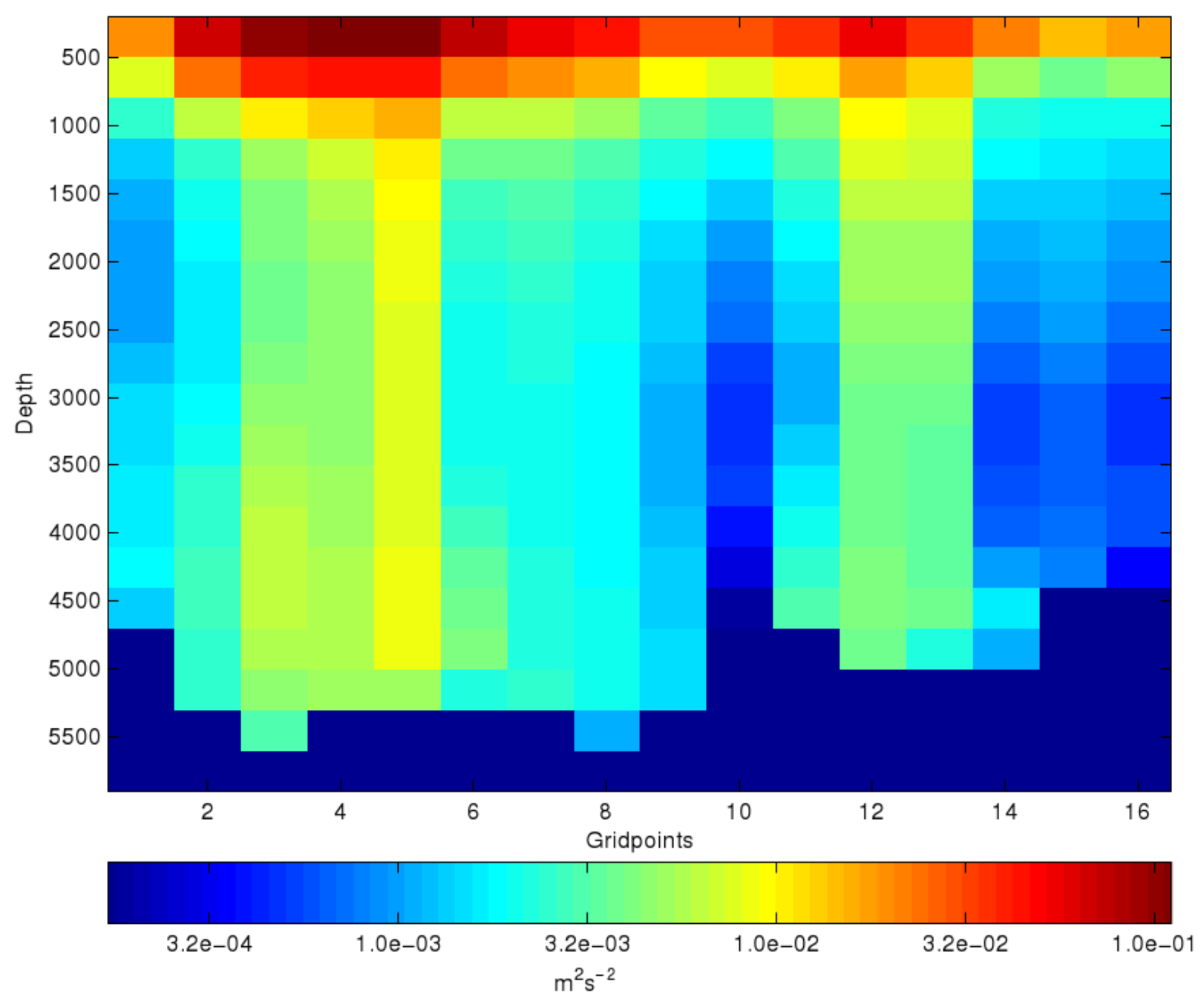


Figure: Eddy kinetic energy of the Eulerian velocity field, calculated from fluctuations around the 5.5 year mean velocity.

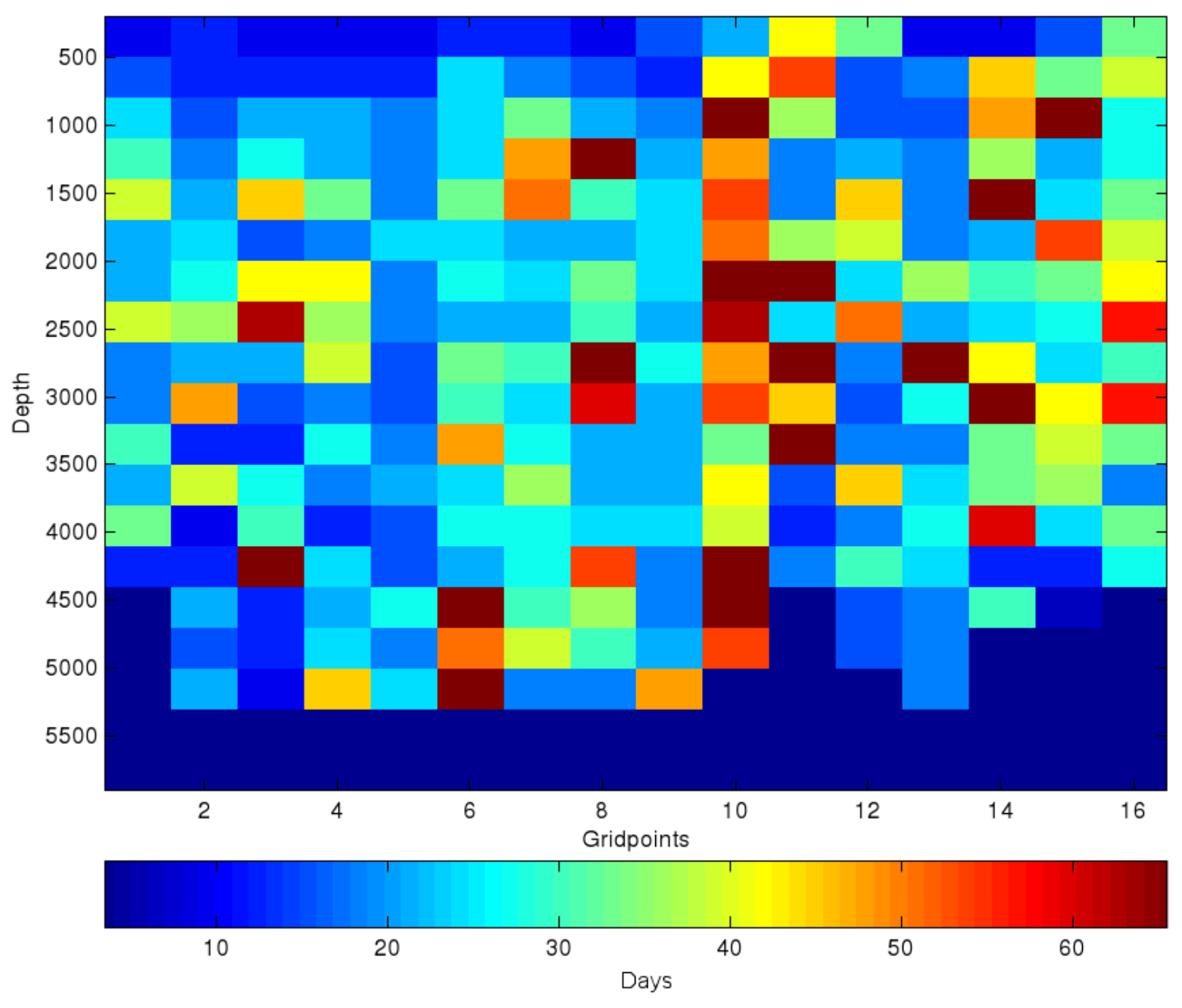


Figure: Timelag at which minor principal component is maximal.

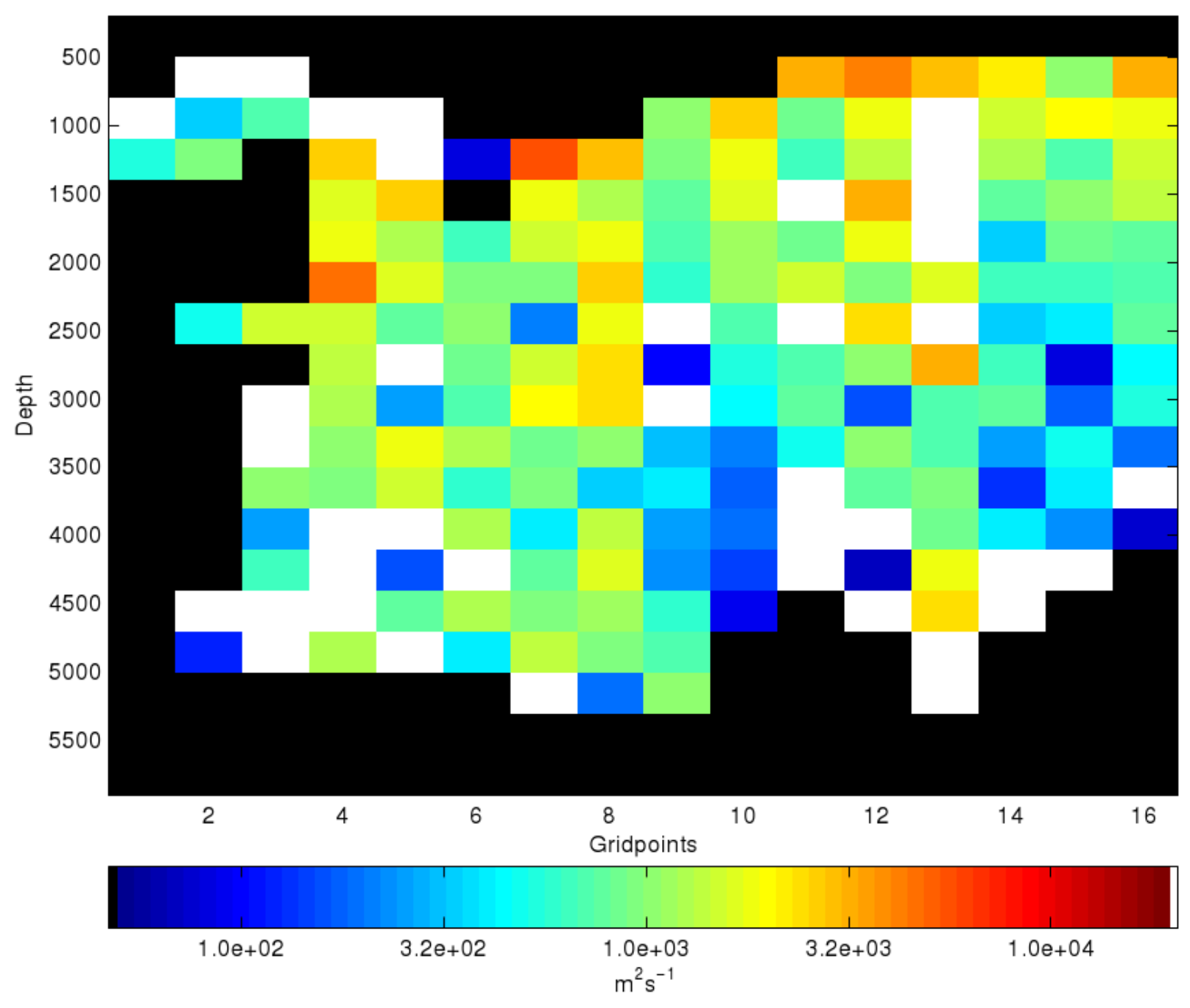


Figure: Minor principal component at 60-day lag. Black boxes represent topography or indicate that drifter mean displacement is not contained within a grid box for a lag of 60 days. Negative values are plotted in white.

Profiles of the diffusivity tensor's maximum value are high above the pycnocline and lower at depth. Below the pycnocline, the value is approximately constant and significantly different from zero. Bootstrap error bars suggest that these structures are significant and that the error is proportional to the magnitude of $max(\kappa_{min}^{sym})$.

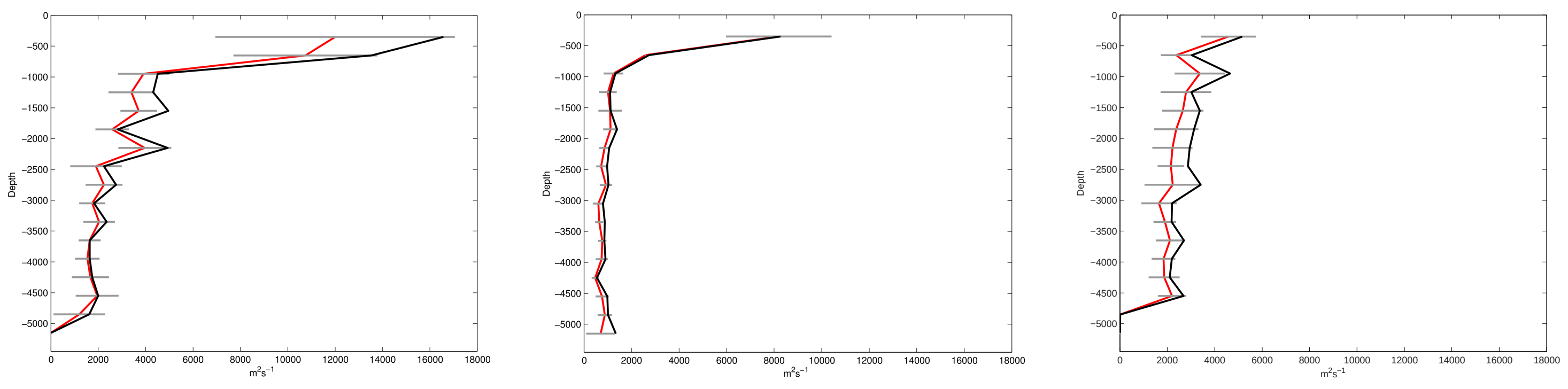


Figure: **Left:** Black: Vertical profile of the minor principal component for gridpoint 4 in the NAC transect. Red: Mean value of bootstrap samples. The pseudo-trajectories in each gridbox were resampled 50 times with replacement. Grey: Standard deviation of bootstrap samples. **Middle:** Gridpoint 9 **Right:** Gridpoint 13

Summary: The maximum of the diffusivity tensor's minor principal component is correlated with EKE, but does not represent an asymptotic value that can be interpreted as a diffusivity.

Future Work

1. Do synthetic drifters follow isopycnals?
2. Under which circumstances is it possible to reach the diffusive limit for κ_{jk}^{sym}?
3. Can we interpret the difference between Eulerian mean flow and Lagrangian mean velocity?
4. Interpretation of κ_{jk}^{asym}: Is there a way to retrieve phase-averaged displacement statistics from isopycnal floats?
5. Full 3-d diffusivity tensor: are displacements orthogonal to isopycnals relevant?
6. Is the major principal component of κ_{jk}^{sym} relevant to parameterizations of shear dispersion in models that cannot resolve narrow mean-flow shear (e. g. western boundary currents)?

References

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