Ice Shelf Calving ~~and Ablation~~: an ice-shelf modeler's perspective on ocean/ice coupling.

Todd K. Dupont
Dept. of Earth System Science
UC Irvine

Collaborators: **R.B. Alley, R. Walker, B.R. Parizek, D. Lindsey, I. Joughin, H.J. Horgan, S. Anandakrishnan, K.M. Cuffey and Jeremy Bassis**

What Is Calving?

- The process by which ice breaks off of the terminus of glaciers and ice shelves
- Usually into water
- Can be from a grounded, but partially submerged, ice cliff
- Or from a floating boundary - a.k.a. an ice front.

Why Look at Calving? Glaciological perspective:

- Calving is the dominant form of ice loss from Antarctica, and about half of loss from Greenland
- Calving rate is a big control on ice-shelf geometry
- Longer ice shelves will buttress outflow of grounded ice more
  - More lateral drag (bigger cork)
  - Greater likelihood for local grounding (ice rises)
- Calving impacts the ice-sheet mass balance and thus sea-level variation

Why Look at Calving? Broader perspective:

- Icebergs are floating sinks for latent and sensible heat in the ocean, and their melting should impact the local salinity (stratification?)
- Evidence suggests a strong relationship between large changes in calving and concomitant climate changes (e.g., Heinrich events)
- Calving dynamics governs the movement of the horizontal boundary between ocean and ice-shelf (i.e., the boundary is not necessarily stationary)

Calving Laws

- Modeling calving “right” is hard - fracture mechanics, on small scales, lots of inputs
- So we attempt to “cheat”, deriving some sort of calving law
- Broadly two types:
  - Calving criteria - dictates where calving will occur - front moves to where criteria is met
  - Calving rate - governs the rate of loss at the front - front moves based on velocity to calving-rate difference

Calving Laws

•Calving criteria examples: magic thickness (e.g. 50 m) or height above buoyancy (Vieli et al., 2002), or crevasse-depth to sea level (Benn et al., 2007), or damage (Pralong and Funk, 2005)

•Calving rate examples: rate follows water depth or height above buoyancy (e.g., Brown et al, 1982 and Sikonia, 1982), or strain-rate

Empirical Calving "Law"?:

- Considering cold, floating termini;
- Looking for the zeroth-order relationship from velocity data
- It would be nice if the relationship depended on variables we already use in models
- Hypothesis: the tendency for ice shelves to fall apart (the near-front spreading rate) controls the rate at which they fall apart (the calving rate).

$c \propto {u_x}^m$

Width and thickness?

$c \propto (HWu_x)^m$

Procedure:

- Assemble ice velocity data, primarily from inSAR
- Measure long. stretching rate about one iceberg-width from the front, especially near center-line of shelf
- Measure “calving rate” (assume s.s. - not so crazy…);
- Plot up the results; do they match the hypothesis?

Calving Law

Whole data set. Positive slope is dominated by Jakobshavn (shown for three different times; J).

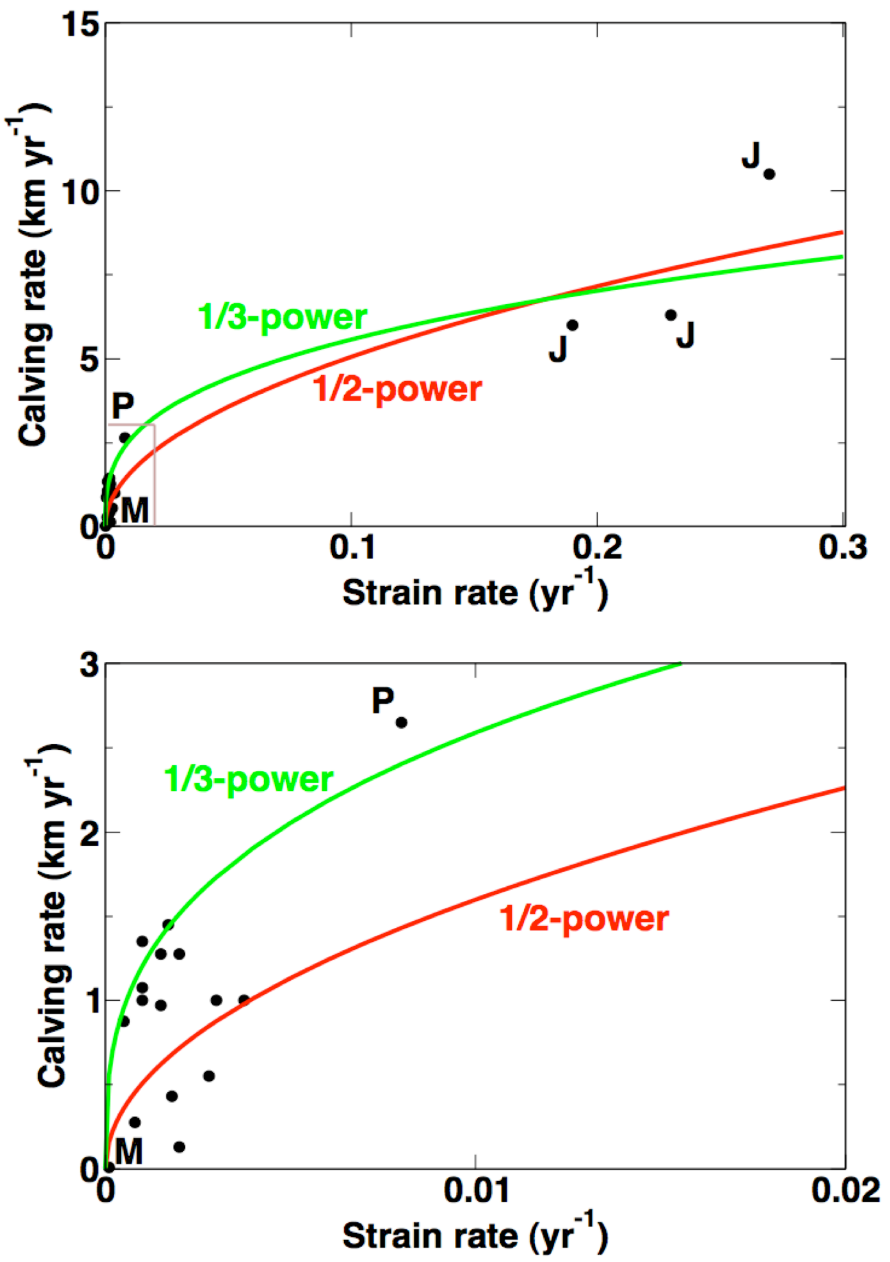
The square-root relation is consistent with fits to various subsets of the data.

Cube root works too.

Plotted line is: $c=1.6x10^{4}*u_{x}^{1/2}$

Explain approx. 90% of the variance

Blow-up of low-strain-rate data. Pine Island (P) and McMurdo (M) dominate. Omitting them leaves a positive-slope relation (noisy, w/ lower confidence).



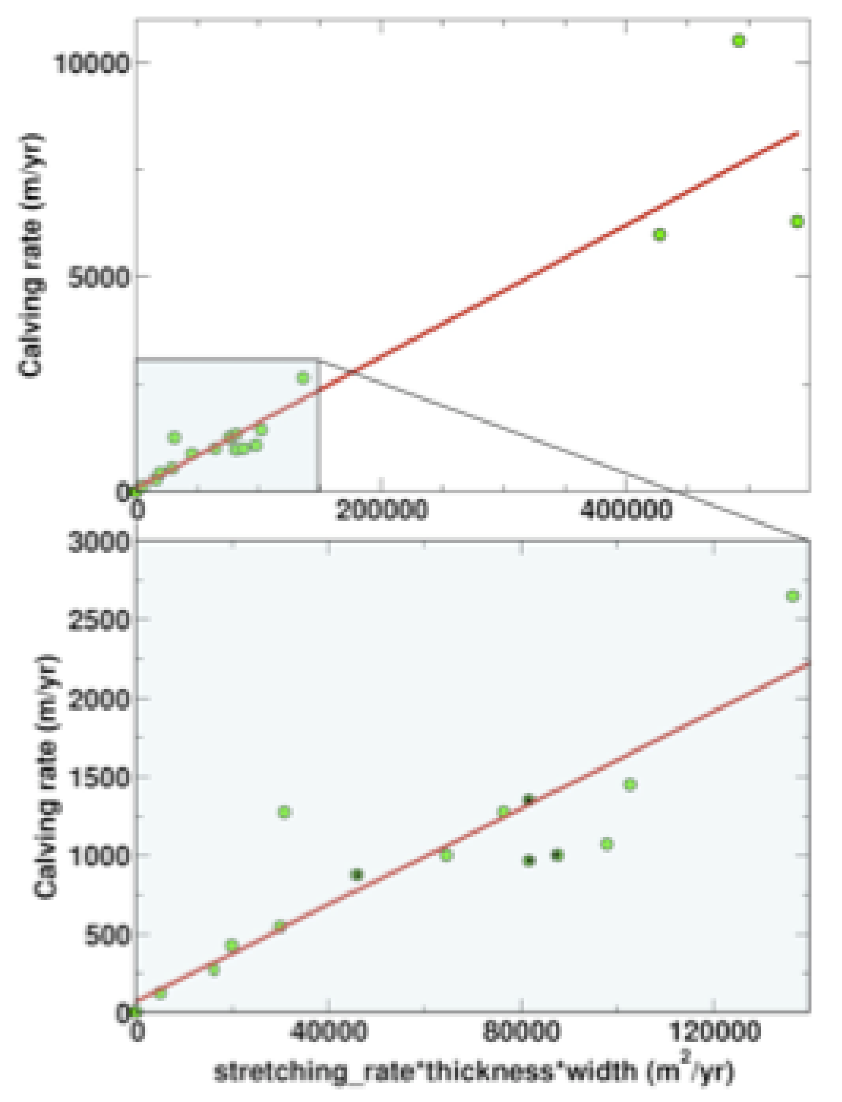
Calving Law - Including Thickness and Width

Intuition and data suggest that thicker and wider ice fronts experience faster calving

Best fit curve is: $c=0.022(Hwu_x)^{0.975}$

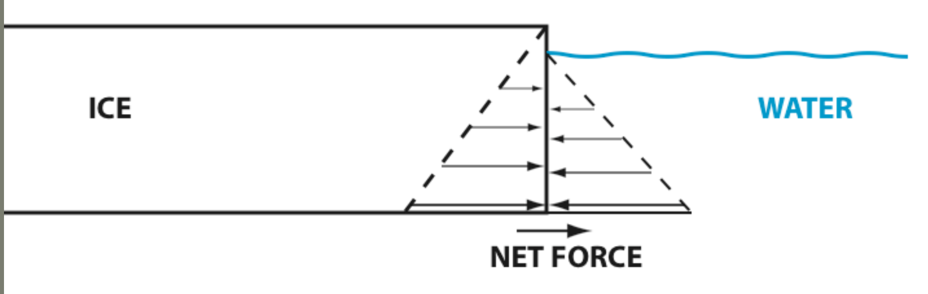
Plotted is: $c=70\ \text{m/yr} + 0.015\ Hwu_x$

Both explain 89% of the variance



Can we use this calving-rate law?

- Limitations of the law
  - empirical correlation (inspired by phys. intuition)
  - noisy
  - essentially 1-d, though generalizing should be easy
  - continuous, not episodic - we won't predict events
  - No water-filled crevasses - no Larsen ice-shelf collapse
  - Won't replicate Jakobshavn - where ice-front torque seems to be important



- But say it's of heuristic value…

Question: What might the <u>dynamic</u> consequences be?

Numerical Experiments

- Implement the calving law in a simplified model of an ice shelf
- Allow the ice front to migrate
- Is there a equilibrium ice front position?
- Is this equilibrium stable or unstable?

Model in brief

1-d, strait-sided (for now), w/ a stretching long. coordinate

$$\eta \equiv \frac{x}{x_{if}(t)}$$

Ice-front balance:

$$\partial_t x_{if} = u_{if} - c = u_{if} - A_c \left(\partial_x u|_{x_{if}} \right)^{\frac{1}{2}}$$

Mass-balance or thickness-evolution equation:

-mapped from t,x to t,η space

-neglects accumulation/ablation (for now)

-bc: const. inlet thickness

$$\partial_t h = \eta \partial_t x_{if} \frac{1}{x_{if}} \partial_\eta h - \frac{1}{x_{if}} \partial_\eta (uh), \quad 0 \leq \eta \leq 1$$

Stress-equilibrium equation:

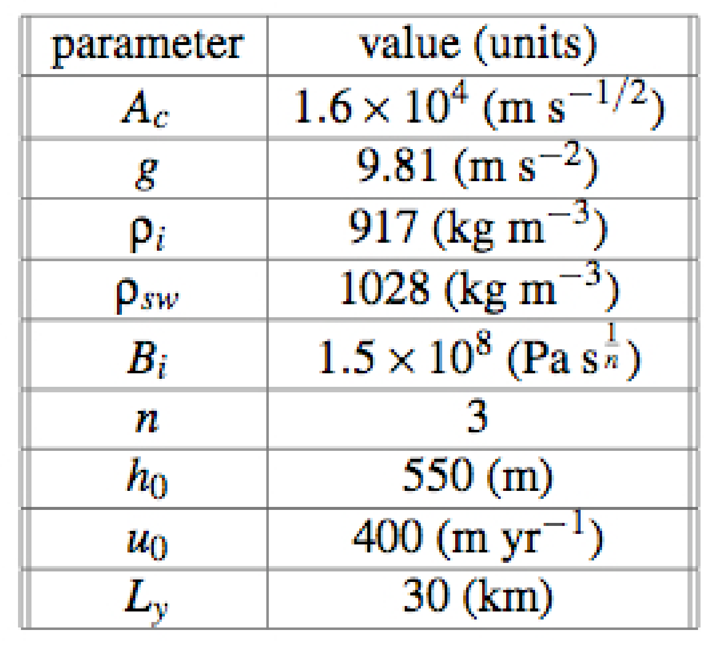
-depth and width-integrated MacAyeal/Morland eqn

-lateral friction treated as boundary-layer phenom.

-bc’s: ice front stretching condition, const. inlet velocity

$$\frac{1}{x_{if}} \partial_\eta \left(4h\nu \frac{1}{x_{if}} \partial_\eta u - \frac{\rho_i g}{2} h^2 \right) = -\frac{\rho_i}{\rho_{sw}} \rho_i g h \frac{1}{x_{if}} \partial_\eta h + \frac{h}{L_y} \gamma_s(u)\, u, \quad \gamma_s \equiv B_s |u|^{\frac{1-n}{n}}$$

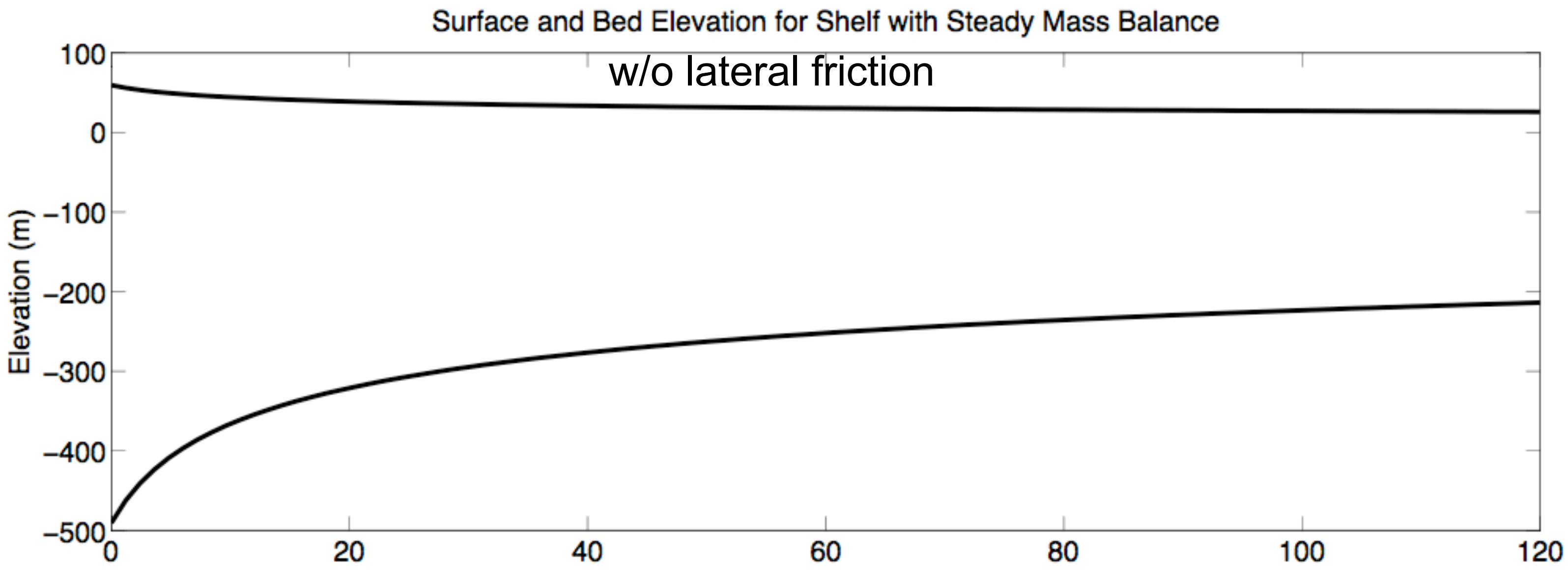
Parameter Values



parameter	value (units)
A_c	1.6×10^4 (m $s^{-1/2}$)
g	9.81 (m s^{-2})
ρ_i	917 (kg m^{-3})
ρ_{sw}	1028 (kg m^{-3})
B_i	1.5×10^8 (Pa $s^{\frac{1}{n}}$)
n	3
h_0	550 (m)
u_0	400 (m yr^{-1})
L_y	30 (km)

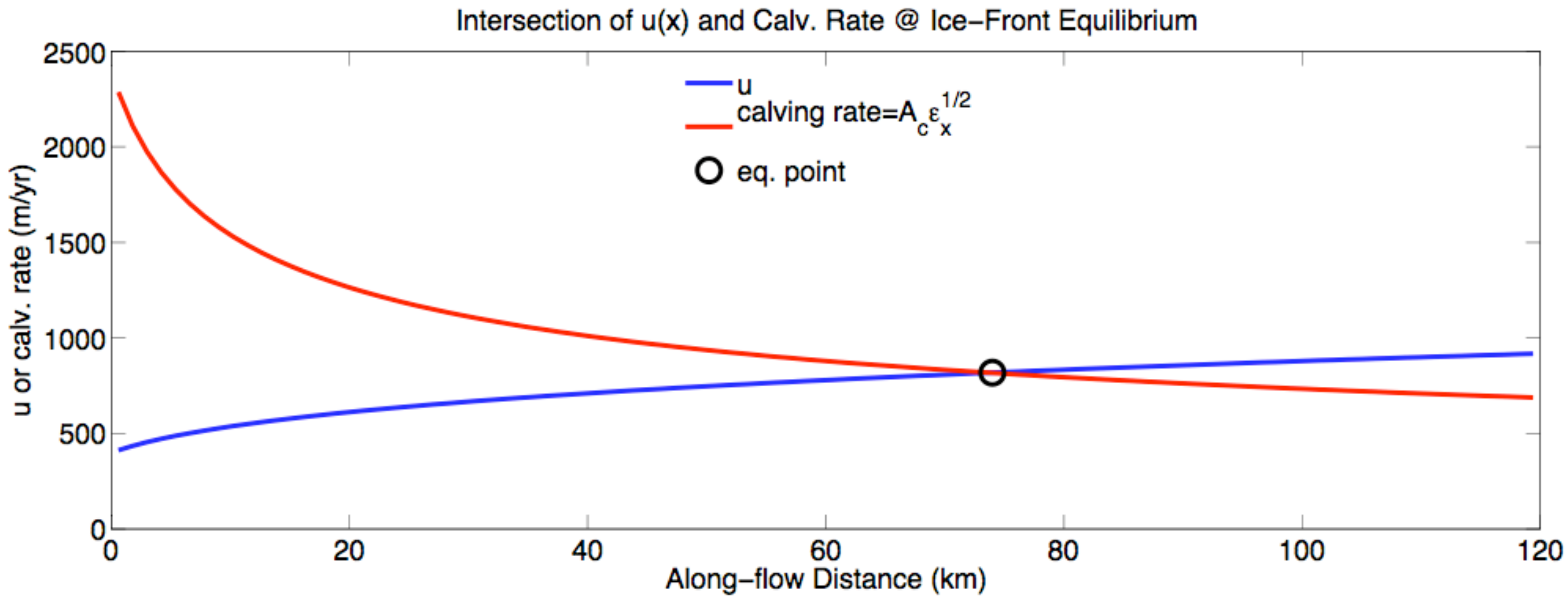
Finding Equilibrium

- For what ice front position is the system at equilibrium?
  - Steady thickness (mass-balance or thickness-evolution eqn)
  - Steady ice front balance
- Straight forward procedure:
  - Hold ice front at a chosen value
  - Let the mass-balance eqn. come to eq.
  - What is the ice-front balance?
  - Change ice front pos. accordingly and iterate
- Easier for shelves w/o lateral friction
  - Plot u_x vs u for a steady (and analytic) profile and see where it crosses the calving-law curve.



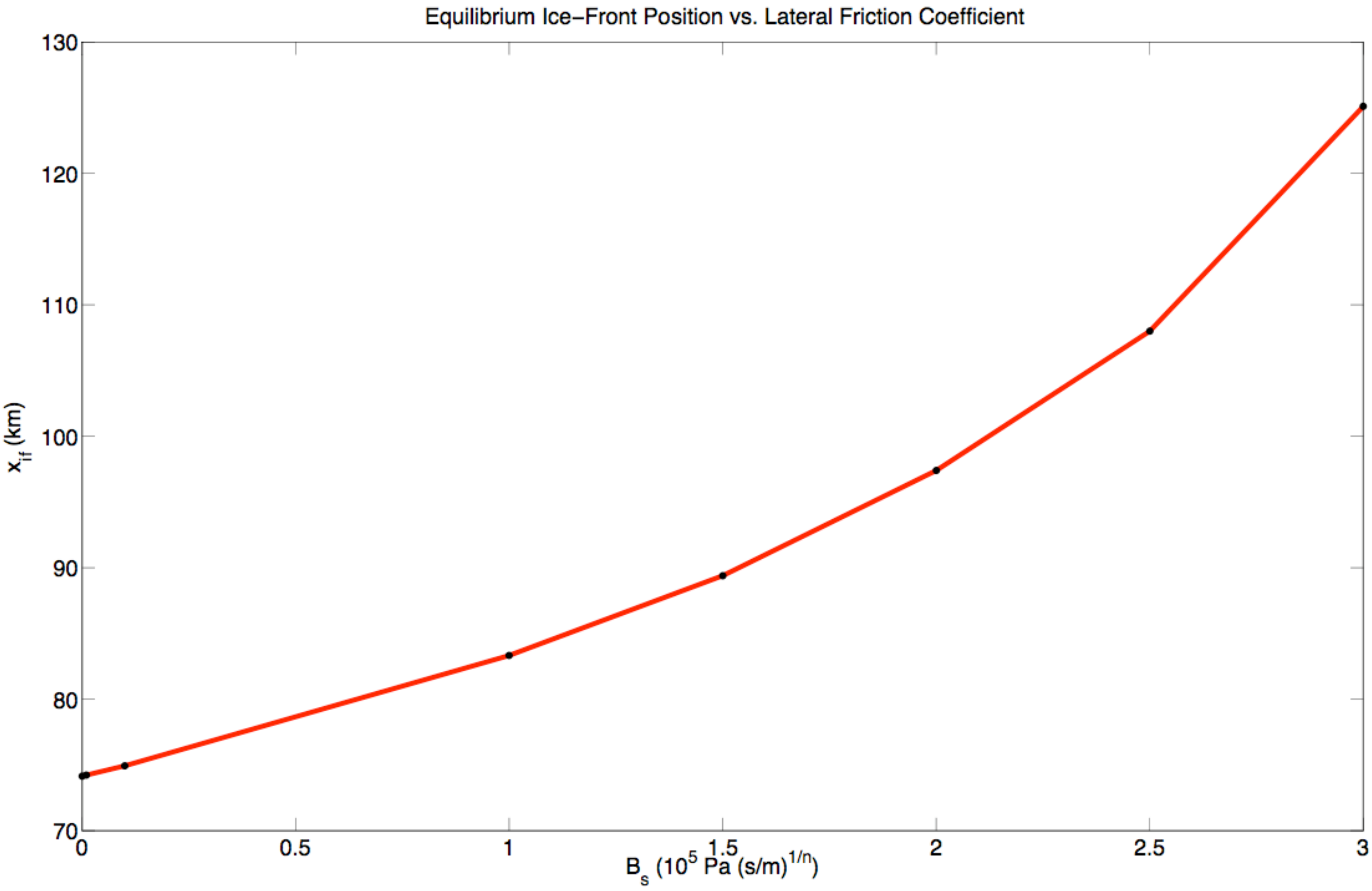
Surface and Bed Elevation for Shelf with Steady Mass Balance

w/o lateral friction



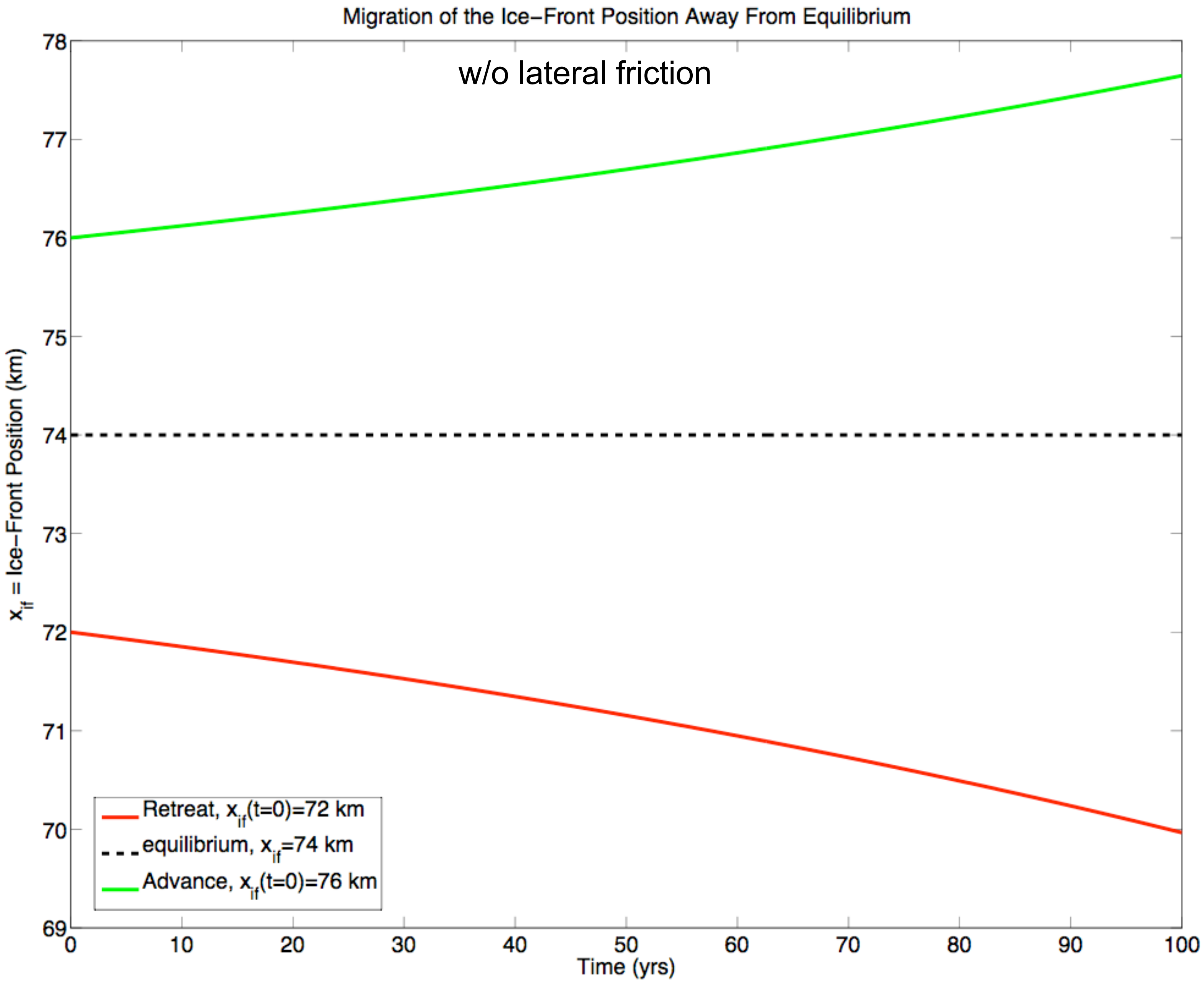
Intersection of u(x) and Calv. Rate @ Ice−Front Equilibrium

Equilibrium Lengthens w/ Lateral Friction



Stability

- Found an ice-front position where transient terms go to zero (equilibrium) w/ and w/o lateral friction
- Question: Is that position stable?
  - Perturb the ice front position from this equilibrium value and see how the system evolves
  - Return to equilibrium position (stable) or no (unstable)



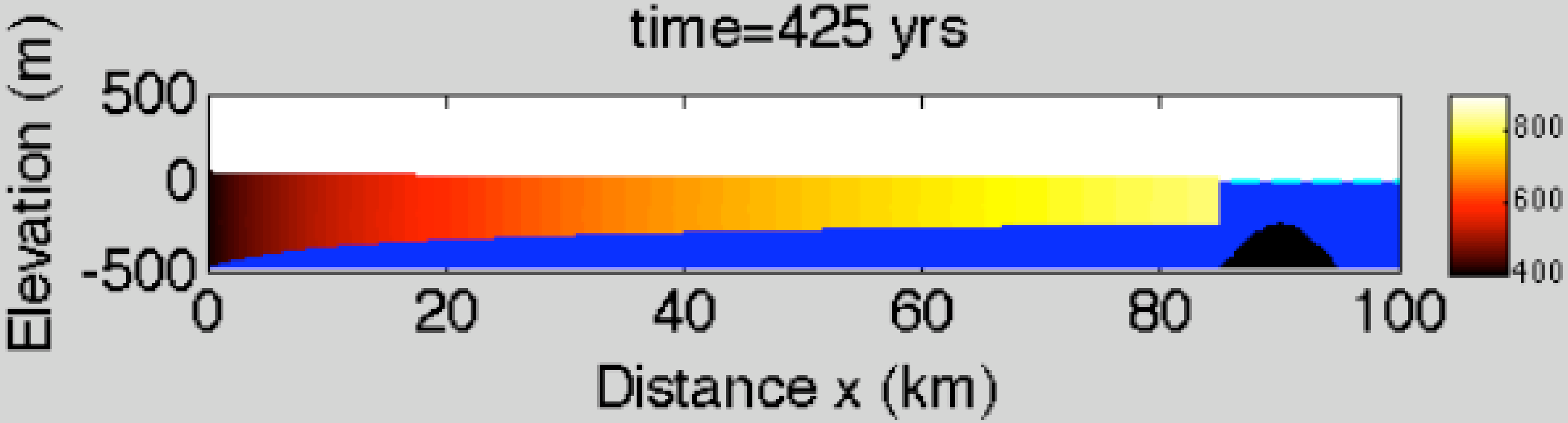
Migration of the Ice−Front Position Away From Equilibrium

w/o lateral friction

Results

- The equilibrium ice-front position is unstable for $c \propto u_x^{1/2}$
- This is also true when lateral friction is included - surprising?
  - Regardless of lateral friction, a retreating ice front is thicker (--> more strain-rate) and slower
- Instability remains w/ thickness and width-dependent calving law Given the apparent quasi-steady positions of real shelves, what's wrong?
  - Law?
  - Implementation?
  - Scenarios? <-- no variable width, no <u>local grounding</u>

Including local grounding

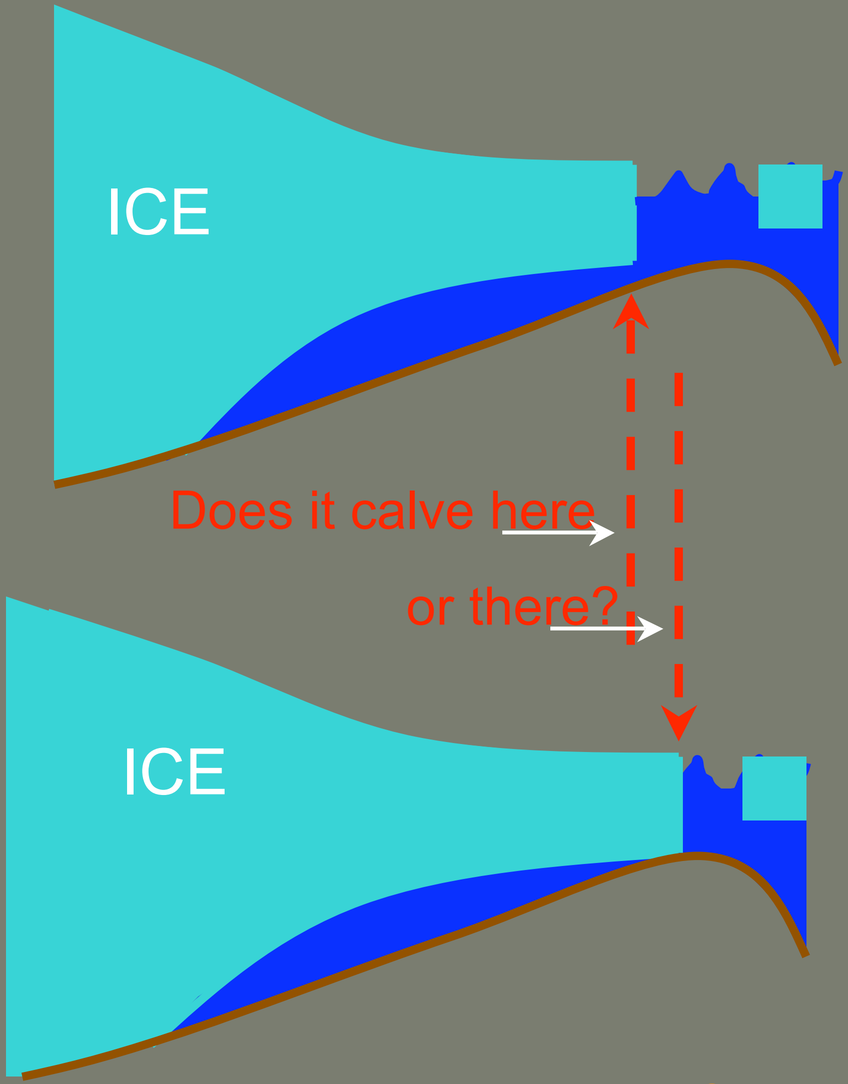


So local shoals can allow for quasi-steady behavior

Concluding Questions

- Will along-flow width variation introduce stability? <-- preliminary exp's say yes
- How do we implement this calving-rate law, or a criterion-based law, in a 2-d or 3-d model? <-- principle strain axes for 2-d?
- Are fixed mesh approaches doomed in the face of a moving boundary? <-- semi-lagrangian easier?
- Do we need to get ice-front melting (ocean/ice coupling!) involved?

Thanks for your attention.



Unbuttressed;
friction from local
high in bed not
stabilizing ice sheet.

Buttressed;
friction from local
high in bed is
stabilizing ice sheet.

To the best of our knowledge, no ice-sheet model calculates this physically. We should do better.